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TECHNICAL NOTE

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FOURIER SERIES OPERATING PACKAGE

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by
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SUMMARY

This report presents a computer program for multiplying, adding, differentiating, integrating, "barring" and scalarly multiplying "literal" Fourier series as such, and for extracting the coefficients of specified terms.

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INTRODUCTION

The Hansen Satellite Theory as modified by Musen[1] involves various manipulations of "literal" Fourier series, as such, before final numerical evaluation. To program this formulation it is necessary to represent cosines, sines, and constants in a special manner and thus manipulate the various Fourier series.

SPECIAL REPRESENTATION OF A SINE, COSINE, OR CONSTANT

It was decided to represent any term, including the constant term, of the Fourier series we are concerned with by two consecutive eight-place floating-point words. The terms of this series have the general form

$$A_n \cos (iF + jE + kW + lU)$$

or

$$A_n \sin (iF + jE + kW + lU)$$

and the values of F, E, W, and U are not used until final numerical evaluation. The first eight-place floating-point word represents the coefficient A_n in an entirely standard manner. The second eight-place floating-point word represents the sine or cosine term of up to four arguments in an *artificial* manner.

[1]Musen, P., "A Modified Hansen's Theory as Applied to the Motion of Artificial Satellites," NASA Technical Note D-492, November 1960.

Consider a term such as A_n cos (iF + jE + kW + lU). Until this term is evaluated we are concerned only with the values of A_n, i, j, k, and l. The coefficient A_n is represented by the first of the two eight-digit floating-point words. The four coefficients of the arguments (i, j, k, and l) are each represented by two digits of the second eight-place floating-point word. The first two digits represent i and are normalized to 00; the next three pairs of digits represent j, k, and l, and are normalized to 50. Thus, the value of i may vary from 0 to 99, while j, k, and l may vary from -49 to +49. A cosine is denoted by a plus sign; a sine by a minus sign, that is, 1 cos (0F + 0E + 0W + 0U) or cos (0) becomes +10000000 + 01, +00505050 + 08. Any constant term can be represented as A_n cos (0). A few additional examples will be helpful (see also Appendix A):

Conventional	Special
1/4 cos (1F + 2E - 3W - 2U)	25000000 + 00, +01524748 + 08
1/4 sin (0F + 1E - 0W + 2U)	25000000 + 00, -00515052 + 08
1/4	25000000 + 00, +00505050 + 08
0	00000000 + 00, +00505050 + 08

The convention of having the first non-zero coefficient of the argument word positive was adopted. Cos(-x) becomes cos(x) and sin(-x) becomes -sin(x). Examples are:

(1/2) cos (0F - 3E + 1W + 0U) becomes (1/2) cos (0F + 3E - 1W + 0U)
and the special representation is 50000000 + 00, + 00534950 + 08

(1/2) sin (0F - 3E + 1W + 0U) becomes -(1/2) sin (0F + 3E - 1W + 0U)
and the special representation is -50000000 + 00 - 00534950 + 08.

The first location address of a series contains the number of terms of the series. A series of n terms would be represented by 2n + 1 words, the first of which would be the number n.

THE FOURIER OPERATING PACKAGE

The following series operations are performed by the Fourier Operating Package:

- Multiplication
- Addition and Subtraction
- Differentiation
- Integration
- Bar (Special operation used in the Hansen satellite theory)
- Scalar Multiplication
- Coefficient Extraction
- Series Evaluation

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Multiplication

Multiplication of two series, where the terms are of the general form described earlier and the values of F, E, W, and U are not used until the final numerical evaluation, is according to the conventional trigonometric identities:

$$A \cos X \cdot B \cos Y = (AB/2) \cos (X+Y) + (AB/2) \cos (X-Y)$$
$$A \cos X \cdot B \sin Y = (AB/2) \sin (X+Y) - (AB/2) \sin (X-Y)$$
$$A \sin X \cdot B \cos Y = (AB/2) \sin (X+Y) + (AB/2) \sin (X-Y)$$
$$A \sin X \cdot B \sin Y = (AB/2) \cos (X+Y) + (AB/2) \cos (X-Y).$$

The Fourier Multiplication routine is composed of three major sections: the multiplier, the collapser, and the arranger.

The Multiplier

Two Fourier series such as

$$A_1A_1^* + A_2A_2^* + A_3A_3^* + \ldots + A_nA_n^* \qquad \text{(Series A)}$$

and

$$B_1B_1^* + B_2B_2^* + B_3B_3^* + \ldots + B_mB_m^* \qquad \text{(Series B)}$$

which are to be multiplied are arranged in descending order of the absolute values of the coefficients, that is,

$$|A_1| > |A_2| > |A_3| > \ldots > |A_n| \text{ and } |B_1| > |B_2| > |B_3| > \ldots > |B_m|$$

To facilitate further discussion, we shall denote any term in the A series $A_xA_x^*$, any term in the B series as $B_yB_y^*$, and any term in the resultant series by $C_zC_z^*$, where A_x, B_y and C_z are the coefficient words and A_x^*, B_y^*, and C_z^* are the argument words.

The multiplication of the A series by the B series proceeds as follows: The first term in the A series is multiplied by each term in the B series, then the second term in the A series is multiplied by each term in the B series, and so on until each term in the A series has been multiplied by each term in the B series. For example, $|A_xB_y|$ is compared with some numerical criterion e. If $|A_xB_y| > e$, then $C_zC_z^*$ and $C_{(z+1)}\, C_{(z+1)}^*$ are generated according to the trigonometric formulas already stated. If $|A_xB_y| \leq e$, then the value of y is examined. If $y > 1$ (i.e., B_y is any term other than the first term), A_x is replaced by $A_{(x+1)}$ and $A_{(x+1)}$ is multiplied by B_1. If $y = 1$, the multiplication of the two series is terminated since any further $A_{(x+1)}\, B_y$ will also be less than e. The multiplication process continues until $A_nA_n^*$ has been multiplied by $B_mB_m^*$ unless the numerical criterion or space limitations intervene.

The Collapser

Every multiplication generates two terms of two words each. The purpose of the collapser is to combine like argument terms. Each argument term C_z^* is compared with each other argument term previously generated and stored. If C_z^* equals any other argument term, the corresponding coefficient terms are added. Thus, there is no duplication of terms.

The Arranger

The final step in the multiplication is the arranging of the terms of the series. $|C_1|$ is compared with $|C_2|$, $|C_3|$, etc. If $|C_z| > |C_1|$, then C_1 is replaced by C_z and C_1^* is replaced by C_z^*. The process continues until the terms are arranged, in descending order, according to the absolute value of the coefficients.

Addition and Subtraction

Addition or subtraction of two Fourier series is primarily a process of comparing argument terms and adding the coefficients of like terms. A_1^* is compared successively with B_1^* through B_m^*, A_2^* with B_1^* through B_m^*, etc., until A_n^* has been compared with B_m^*. If $A_x^* = B_y^*$, the sum of the coefficients $(A_x + B_y)$ and the argument term A_x^* are stored, and B_y and B_y^* are replaced by zeros. If A_x^* does not equal any B_y^*, both A_x and A_x^* are stored. After all terms in the A series have been compared with all terms in the B series, the remaining B series terms are stored.

Subtraction is accomplished in like manner after changing the signs of each coefficient term in the B series. The resultant series in each case is processed through the arranger.

Differentiation

Differentiation, in this application, is with respect to the F variable. Thus,

$$\left(\frac{\partial}{\partial F}\right) A \sin (iF + jE + kW + lU) = iA \cos (iF + jE + kW + lU).$$

Example:

$$\left(\frac{\partial}{\partial F}\right) \sin (3F + 2E - 3W + U) = +3 \cos (3F + 2E - 3W + U)$$

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and

$$\left(\frac{\partial}{\partial F}\right)[10000000 + 01, - 03524751 + 08 \text{ becomes } +30000000 + 01, +03524751 + 08.$$

On completion of the differentiation, the resultant series is processed thru the arranger.

Integration

Integration, in this application, is with respect to E. However, W is also a function of E. Thus,

$$\int A \cos (iF + jE + kW + lU)dE = \frac{A}{c_1 j + c_2 k} \sin (iF + jE + kW + lU).$$

<u>Example</u>:

With $c_1 = 1$ and $c_2 = 1$,

$$\int[30000000 + 01, + 01525250 + 08]dE \text{ becomes } 75000000 + 00, -01525250 + 08.$$

The resultant integrated series is also processed through the arranger.

Bar Operation

The bar operation is a special function in the Hansen Satellite Theory. It consists of adding the coefficient of the F argument to the coefficient of the E argument and substituting zero for the F coefficient. Thus,

$$A \cos (iF + jE + kW + lU) \text{ after barring becomes } A \cos (0F + (i + j) E + kW + lU).$$

<u>Example</u>:

50000000 + 00, + 02534850 + 08 after barring becomes 50000000 + 0C, + 00554850 + 08.

Scalar Multiplication

Scalar multiplication is the multiplication of the coefficient A_n of each term by a constant.

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Coefficient Extraction

In the Hansen Satellite Theory it is sometimes necessary to use the coefficient of some term of a Fourier series such as a sine 1F term, cosine 2E term, or the constant term of a series. Let us assume it is necessary to use the constant term. If there is a constant term in that series, it will be the multiplier of the cos (0), or in special form, that A_n word which multiplies 00505050 + 08. We successively compare each argument term in the series with cos (0) and extract that A_n which multiplies the argument term cos (0). If no argument term of the series is cos (0), a normalized zero (10000000 + 00 + 00000000 + 00 + 00505050 + 08), is printed.

Series Evaluation

The numerical values F, E, W, and U are only employed in the Series Evaluation Routine.

To evaluate a Fourier series, the numerical values of i, j, k, and l are multiplied by the numerical values of F, E, W, and U, and the sum iF + jE + kW + lU is determined. The sine or cosine of iF + jE + hW + lU is multiplied by the coefficient A and the terms are added.

CONCLUDING REMARKS

Appendix A is the special representation of a nine term series. Appendix B presents flow charts and Appendix C a listing of instructions for the program packages for multiplication (including the collapser and arranger), addition or subtraction, differentiation, integration, bar operation, scalar multiplication, coefficient extraction, and the evaluation of the final series. Because this program was written in Mystic Code for the IBM 709, an explanation of Mystic Code is given in Appendix D.

The Fourier operating package can be used with any theory that involves representations of functions by Fourier series. It can also be modified to operate with polynomials of the form $X^aY^bZ^cU^d$.

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ACKNOWLEDGMENTS

The author is indebted to Messrs. R. G. Kelly and T. P. Gorman for their aid in the construction of the package and for the translation into Mystic Code and to Aileen Marlow for preparing the flow charts.

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Appendix A

Nine Term Series in Standard and Special Form

The following is a nine term series presented in standard and in special form. Note that in the special form, the first location (address) contains the number of terms in the series.

Series in standard representation

```
 .29467121 cos (0)
+.00010496334 cos (0F + 0E + 2W + 0U)
+.00005252596 sin (0F + 1E - 1W + 0U)
+.000019845618 cos (0F + 1E - 2W + 0U)
+.0000066329604 cos (0F + 1E + 2W + 0U)
+.0000020107054 cos (0F + 2E - 2W + 0U)
-.00000036004597 sin (0F + 0E + 1W + 0U)
-.000000055052357 sin (0F + 0E + 3W + 0U)
-.000000031090653 sin (0F + 1E + 1W + 0U)
```

Series in special representation

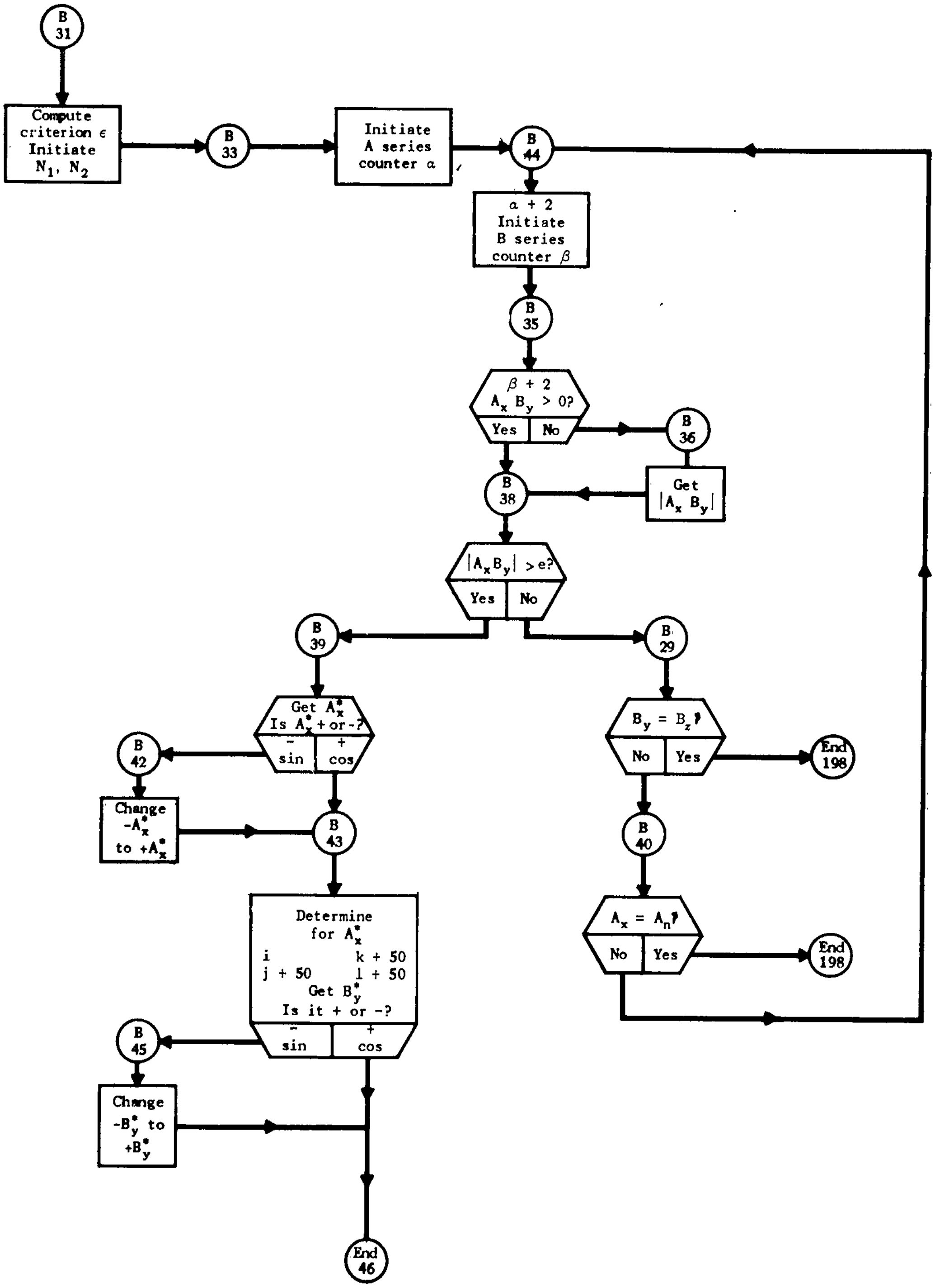
```
 90000000 + 01
+29467127+00, +00505050+08
+10496334-03, +00505250+08
+52525962-04, -00514950+08
+19845618-04, +00514850+08
+66329604-05, +00515250+08
-20107054-05, +00524850+06
-36004597-06, -00505150+08
-55052357-07, -00505350+08
-31090653-07, -00515150+08
```

Appendix B

Flow Charts

The following are the flow charts for the operating packages for multiplication (including the collapser and the arranger), addition or subtraction, bar operation, differentiation, integration, and series evaluation.

Flow Chart for Multiplication



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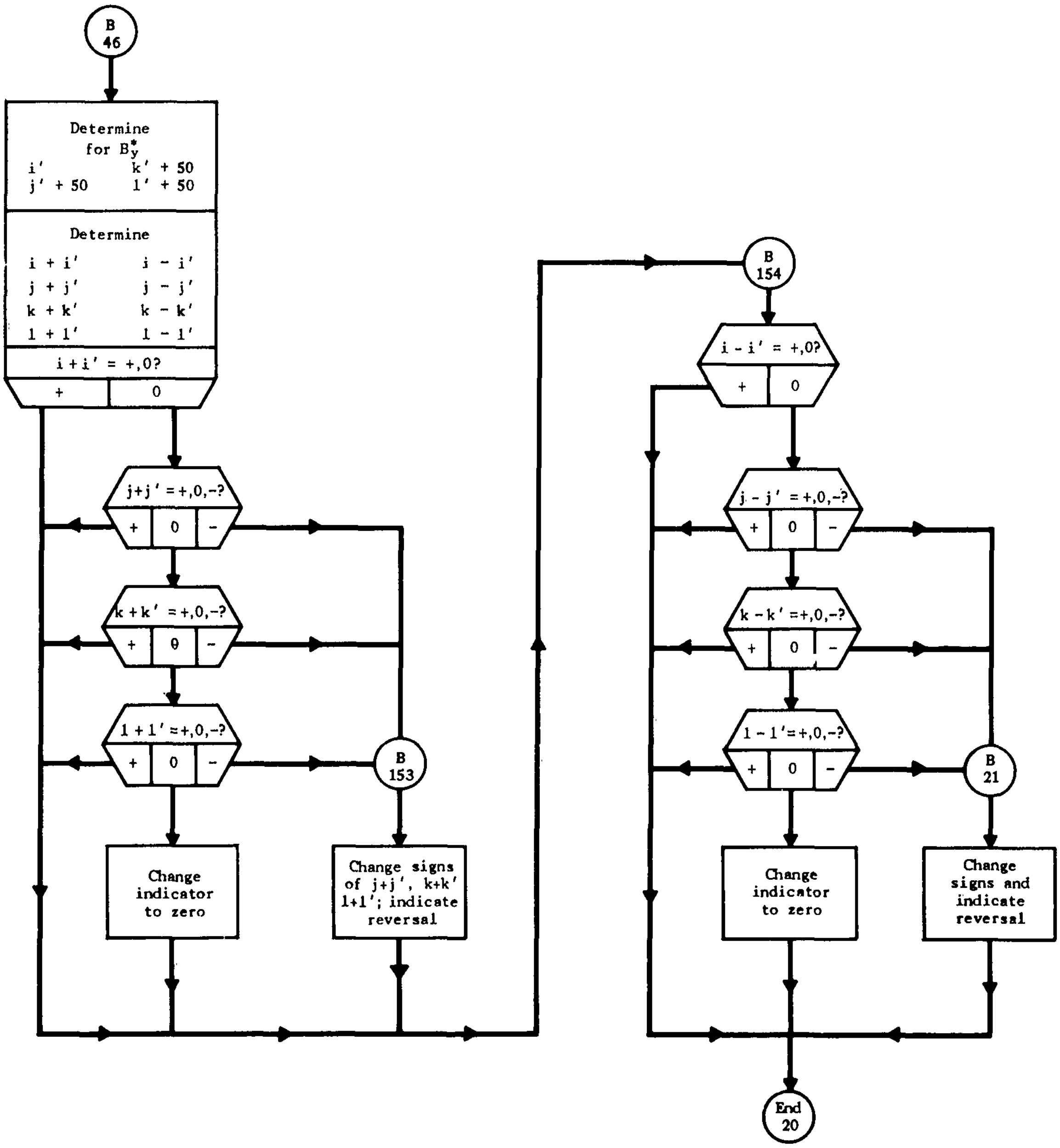
Flow Chart for Multiplication (Continued)



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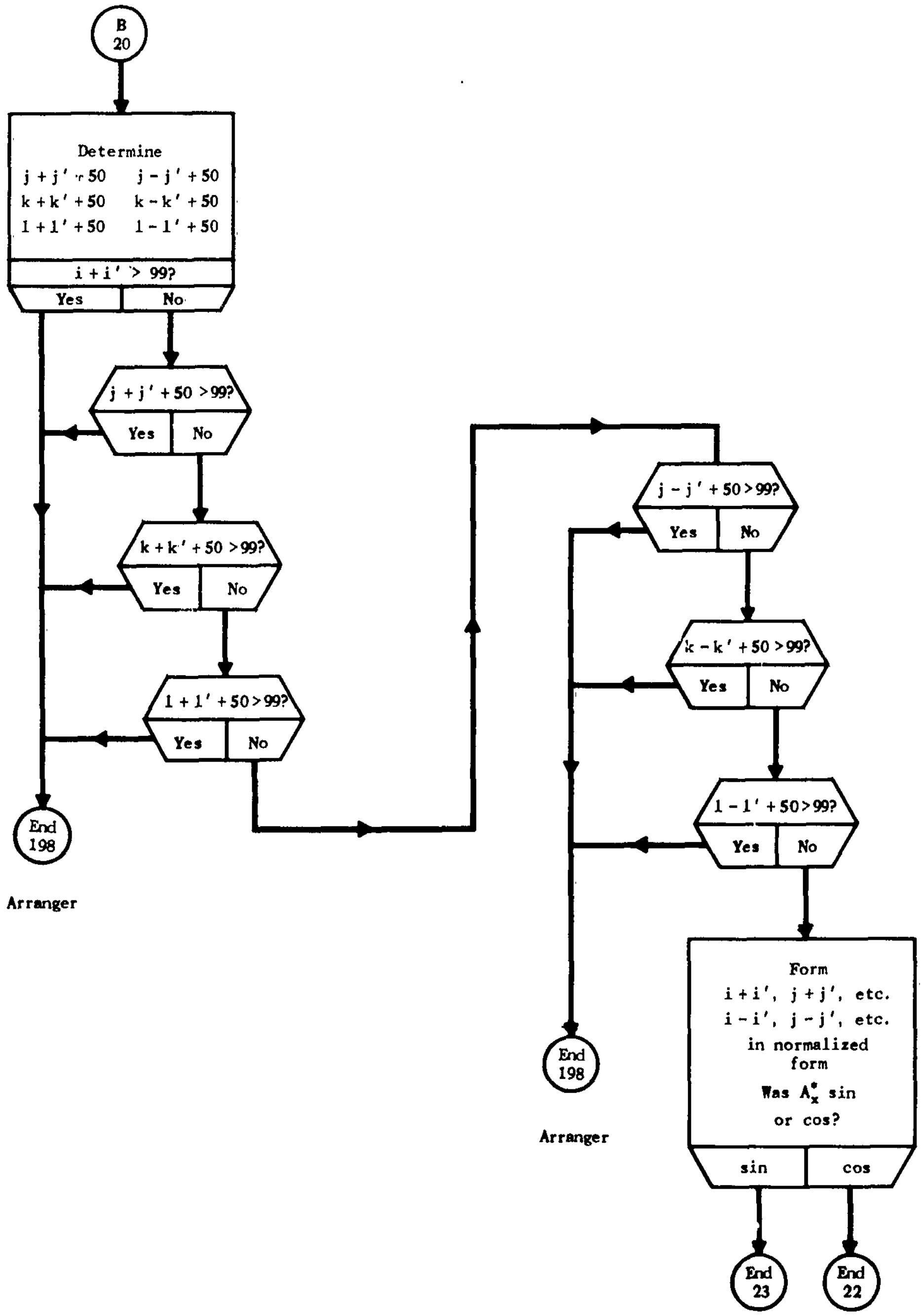
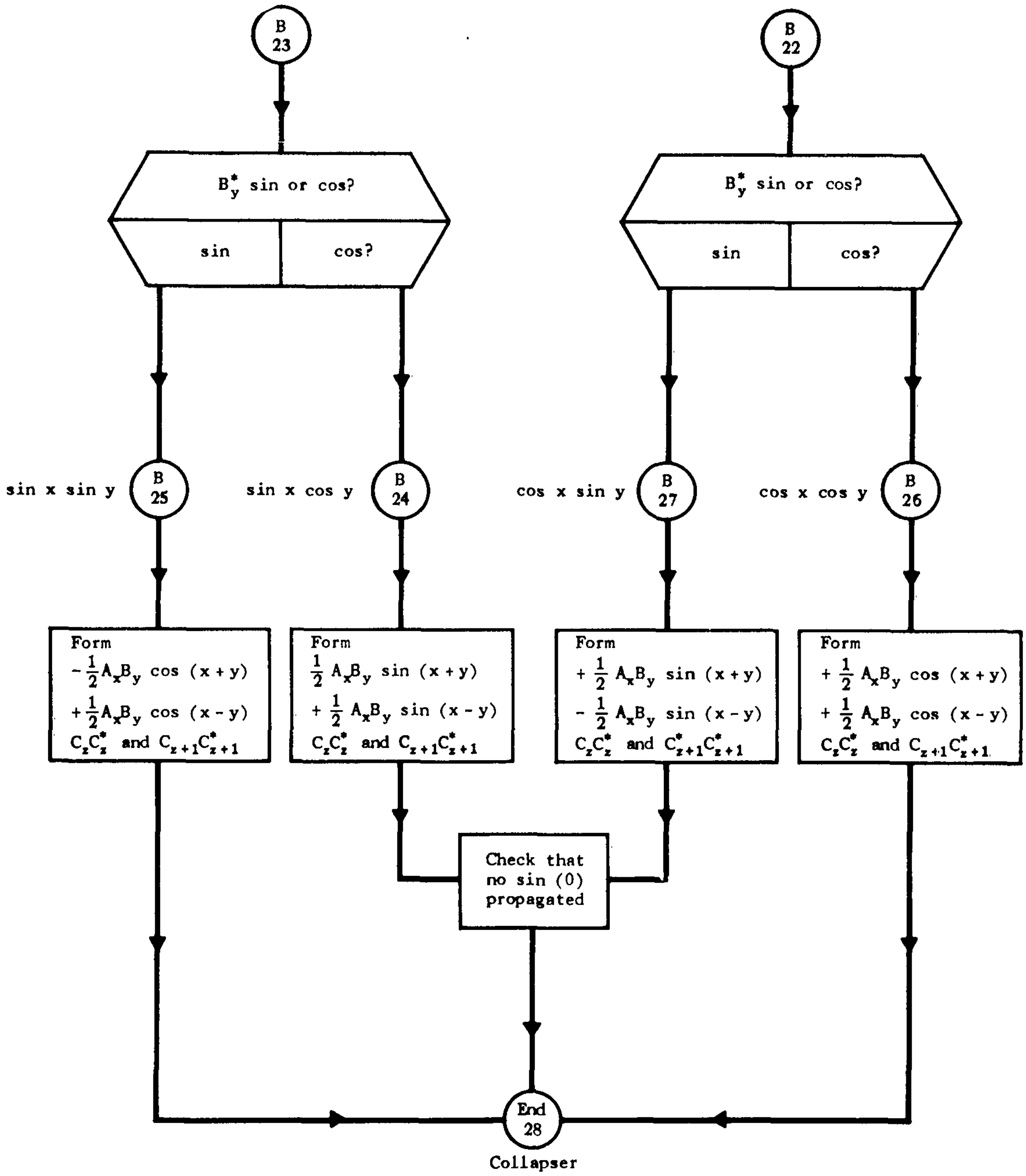
Flow Chart for Multiplication (Continued)



Flow Chart for Multiplication (Continued)



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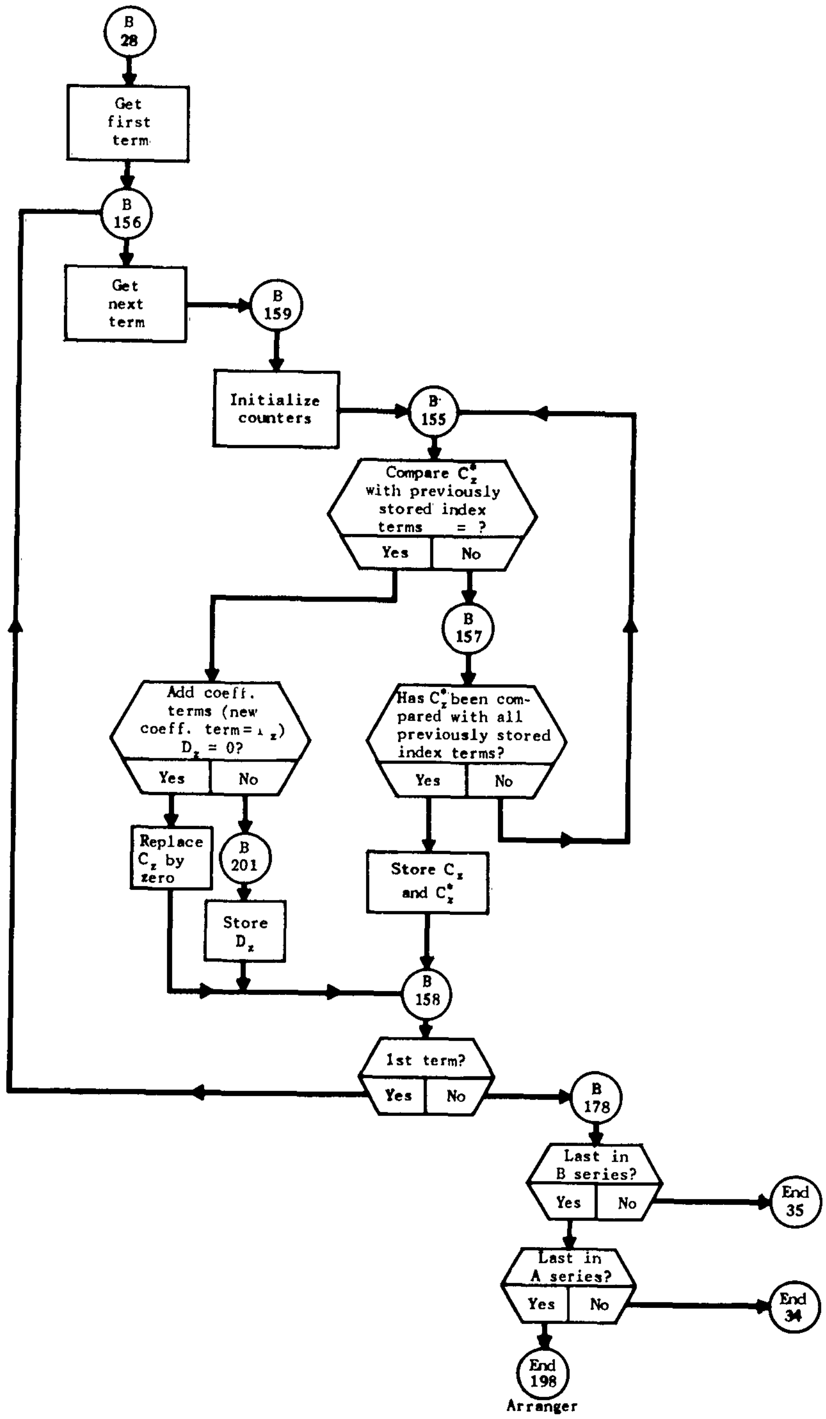
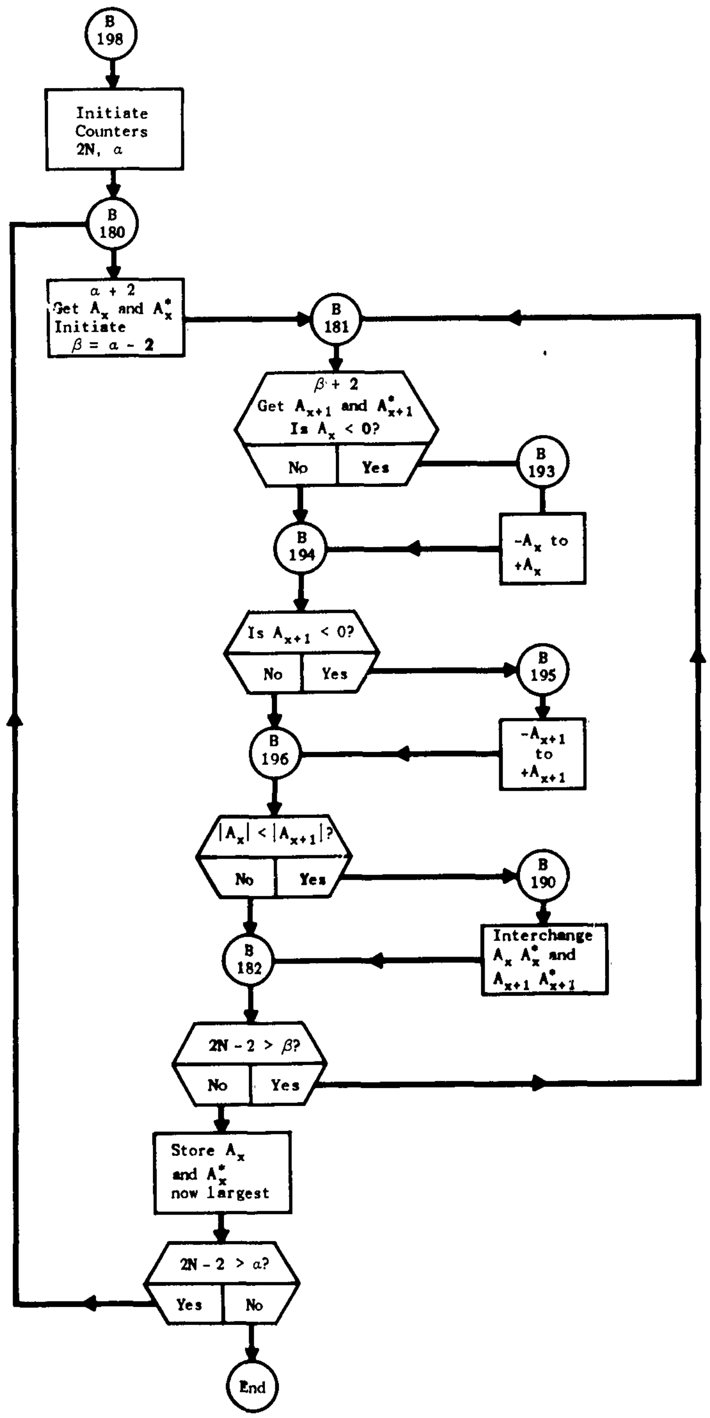
Flow Chart for Multiplication (Continued)

Collapser



Flow Chart for Multiplication (Continued)

Arranger



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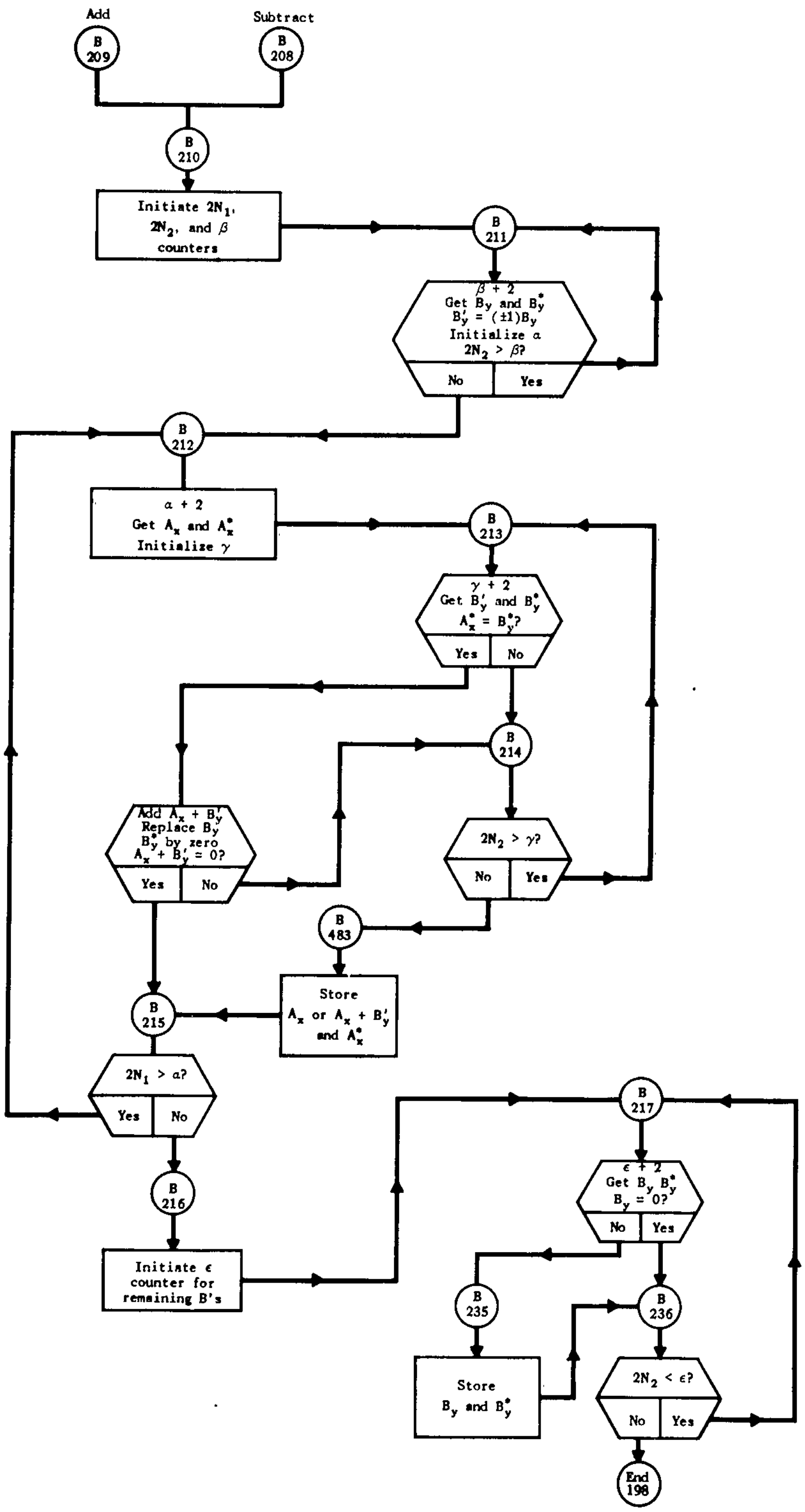
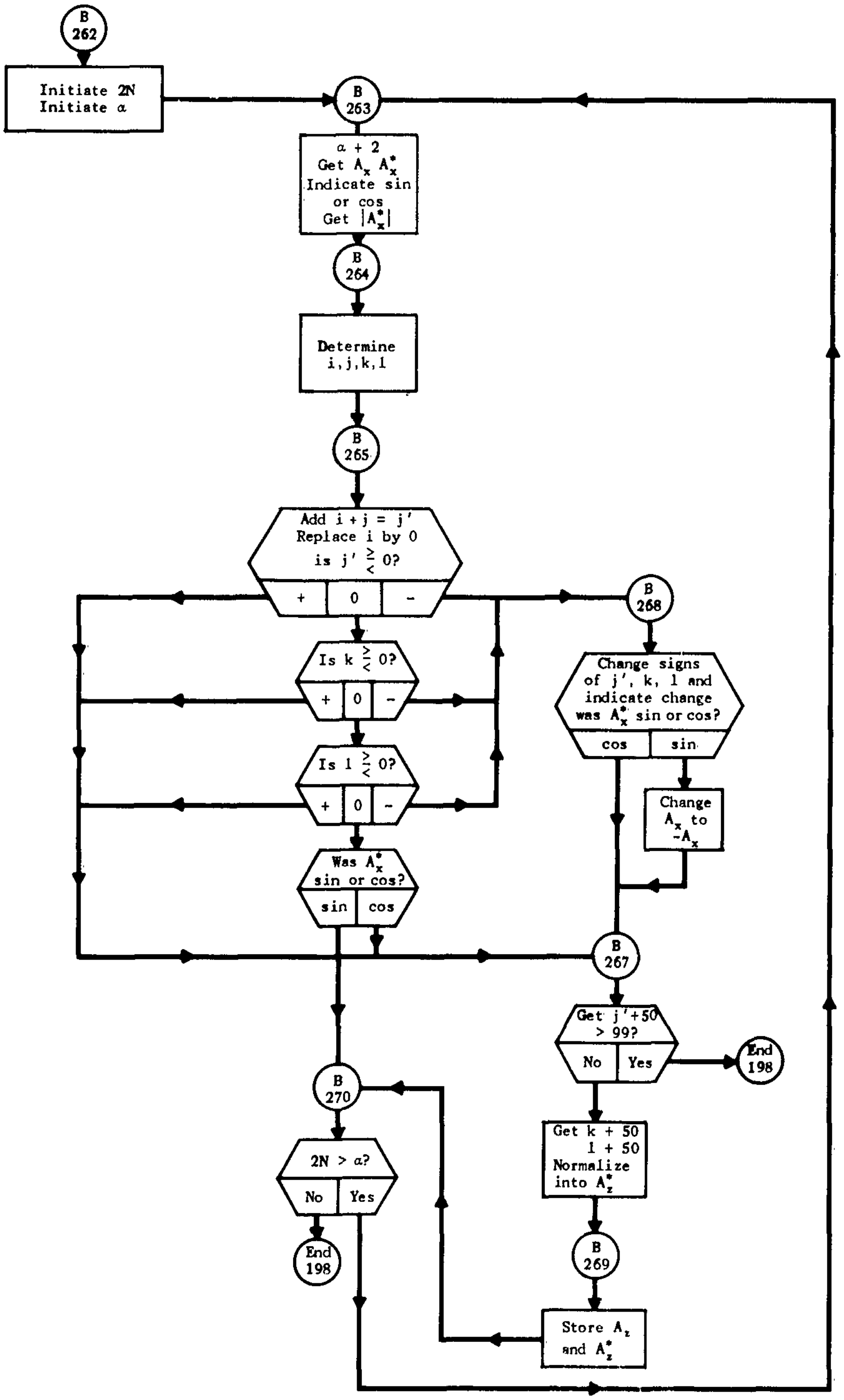
Flow Chart for Addition or Subtraction



Flow Chart for Bar Operation



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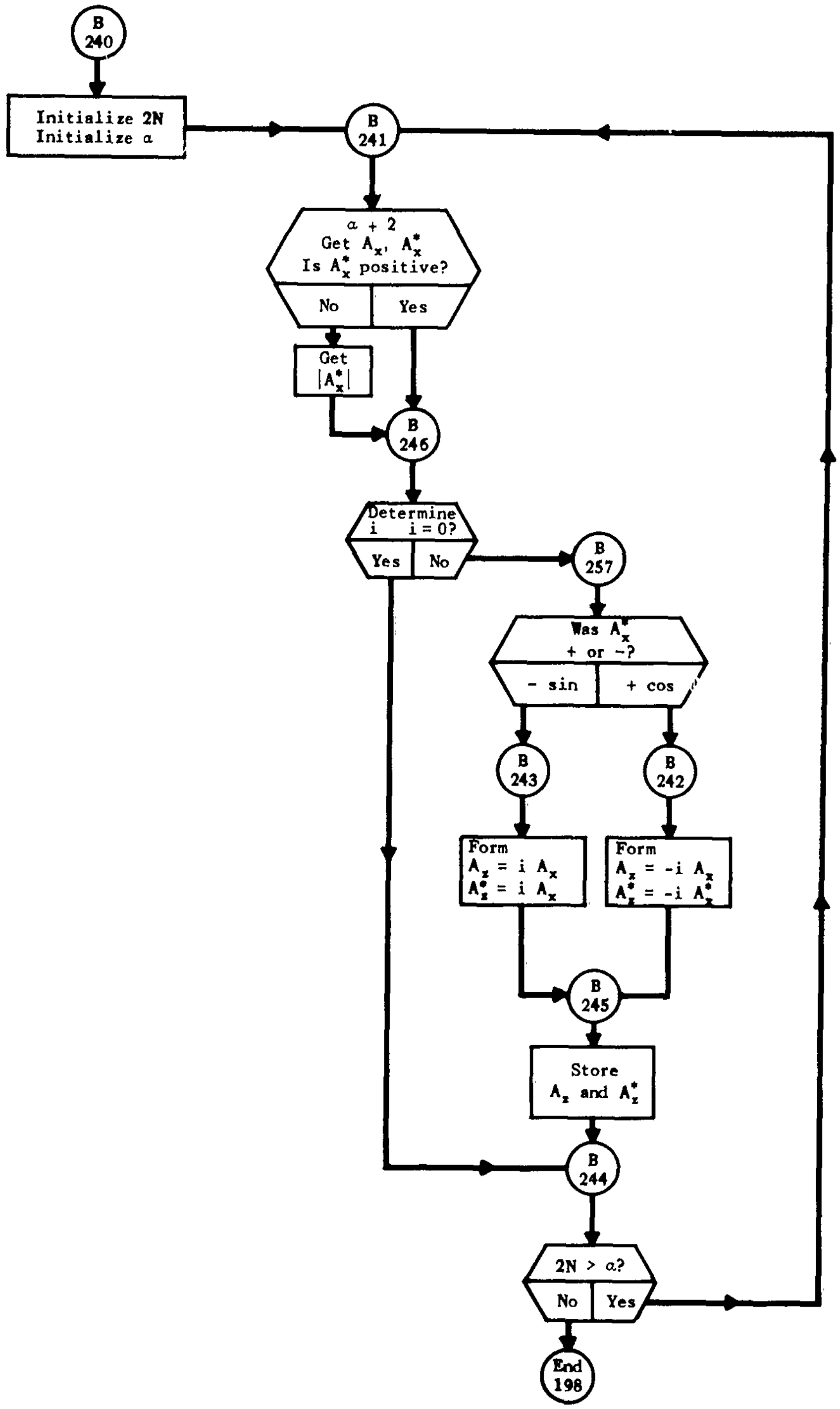
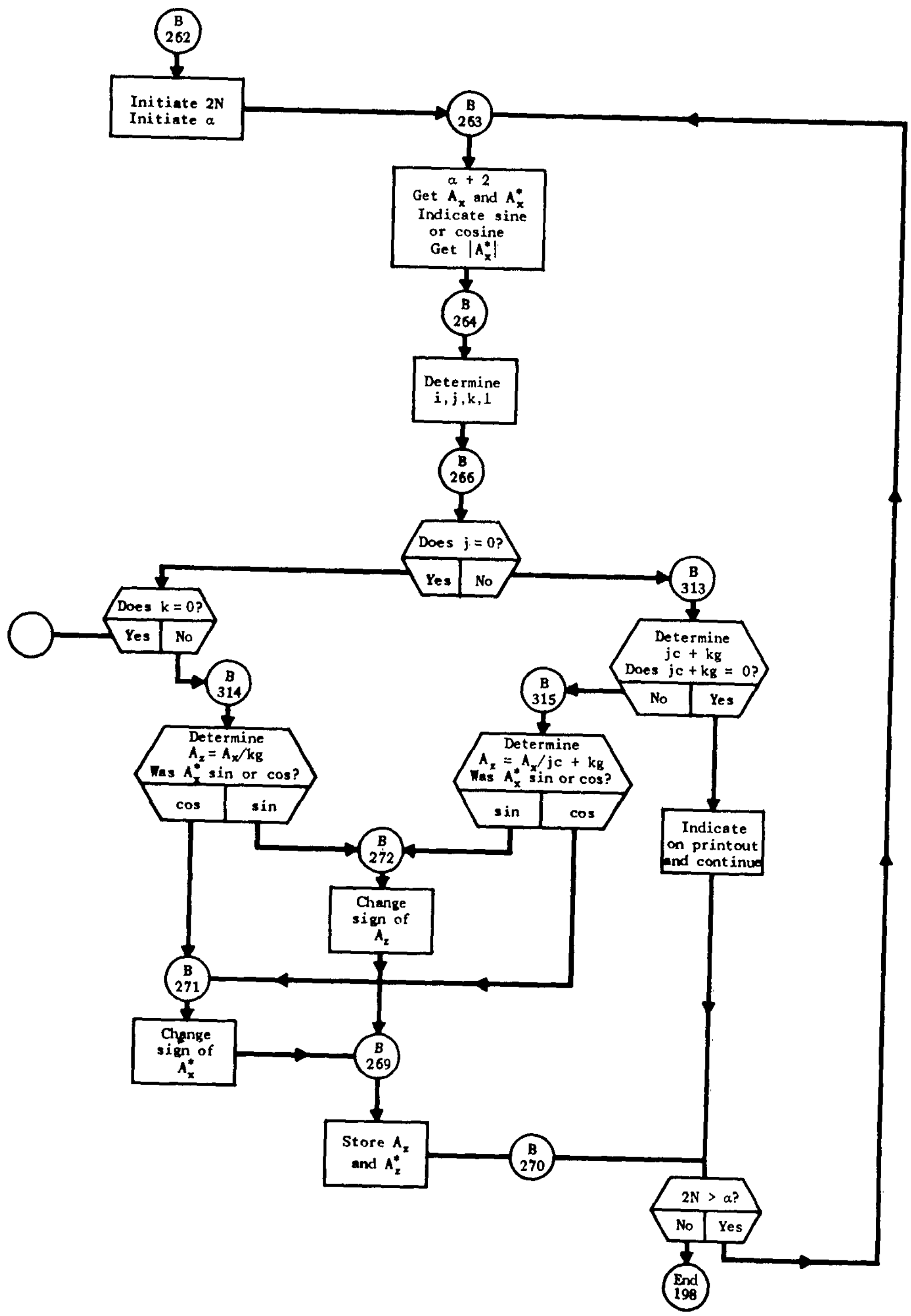
Flow Chart for Differentiation



Flow Chart for Integration



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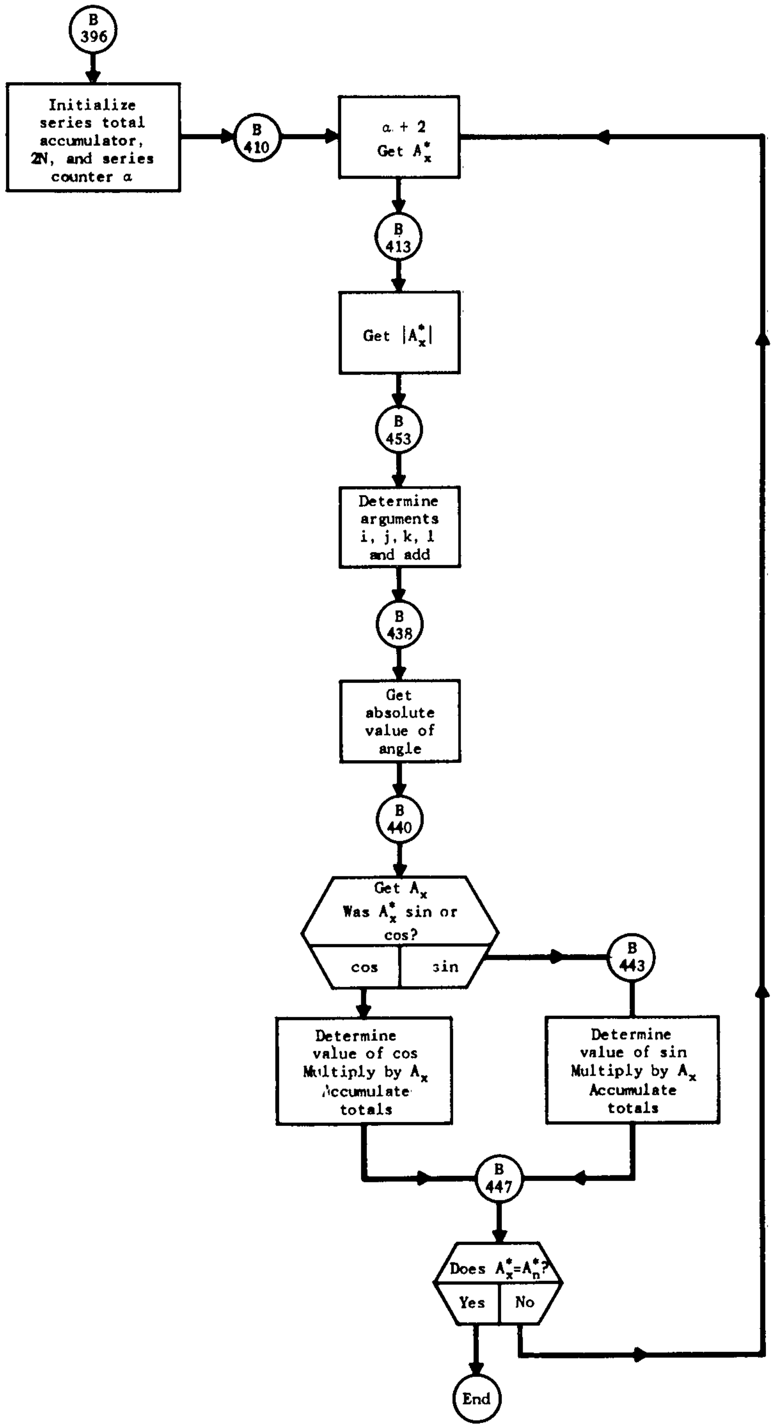
Flow Chart for Series Evaluation



Appendix C

Listing of Instructions

The following is a listing of instructions for multiplication (including the collapser and the arranger), addition or subtraction, differentiation, integration, bar operation, scalar multiplication, and coefficient extraction.

The square root instructions are for standard number representations and therefore have not been discussed in the report.

```
                                  K=   00000

       HANSEN SATELLITE THEORY MLC
             ADAPTER PACKAGE
 @ 21750                                ORIGIN CARD
 K 00000
 K 00050
*B 00001                                      SQUARE ROOT FUNCTION
 V 00005 +00000000+00
 V 00007  10000000 01
 V 00010  20000000-07
 G 00003 00001 00003
 C 00003 00005 00006
 H 00001 00004 00005
 E 00002
*B 00006
 A 00008 00007 00007
 R 00009 00007
 S 00011 00007 00010
 C 00007 00003 00012
 R 00009 00003
*B 00012
 D 00013 00003 00009
 A 00014 00009 00013
 D 00015 00014 00008
 D 00016 00015 00009
 R 00009 00015
 C 00011 00016 00012
 H 00001 00004 00009
 E 00002
 K 00000
 K 00070
*B 00001                                      SIN COSINE FUNCTION
 V 00009  17000000-04
 V 00010  10000000 01
 V 00011  67108864 08
 V 00012  15707963 01
 V 00013 -64596371 00
 V 00014  79689679-01
 V 00015 -46737660-02
 V 00016  15148400-03
 V 00018 +00000000+00
 V 00019 +62831853+01
 G 00003 00001 00003
```

K= 00070

```
 R 00020 00010              043
 C 00003 00018 00021        044
 S 00020 00018 00020        045
 S 00003 00018 00003        046
*R 00021                    047
 D 00022 00003 00019        048
 A 00022 00022 00011        049
 S 00022 00022 00011        050
 M 00022 00022 00019        051
 S 00003 00003 00022        052
 M 00003 00003 00020        053
 S 00020 00018 00012        054
 R 00023 00010              055
*B 00024                    056
 I 00025 +31415926+01       057
 C 00003 00012 00026        058
 S 00025 00018 00025        059
 C 00020 00003 00026        060
 D 00003 00003 00012        061
 M 00027 00003 00003        062
 M 00028 00016 00027        063
 A 00028 00028 00015        064
 M 00028 00028 00027        065
 A 00028 00028 00014        066
 M 00028 00028 00027        067
 A 00028 00028 00013        068
 M 00028 00028 00027        069
 A 00028 00028 00012        070
 M 00028 00028 00023        071
 M 00003 00028 00003        072
 H 00001 00004 00003        073
 F 00002                    074
*R 00026                    075
 S 00003 00003 00025        076
 S 00023 00018 00023        077
 F 00024                    078
*R 00005                    079
 G 00003 00005 00007        080
 S 00017 00012 00003        081
 F 00017 00001 00017        082
 H 00005 00008 00017        083
 F 00006                    084
```

```
                                                                     PAGE003
                          K=   00070

 K 00000                                                              085
 K 00100                                                              086
*B 00001                   FØURIER SERIES MØVE                        087
 G 00005 00001 00003                                                  088
 A 00006 00005 00005                                                  089
 A 00006 00006 00003                                                  090
 H 00001 00004 00005                                                  091
*B 00007                                                              092
 A 00003 00003 00008                                                  093
 A 00004 00004 00008                                                  094
 G 00005 00001 00003                                                  095
 H 00001 00004 00005                                                  096
 C 00006 00003 00007                                                  097
 F 00002                                                              098
 V 00008 +10000000+01                                                 099
 K 00010                                                              100
*B 00001                   FØURIER ADDITIØN (11)                      101
 R 00080 00299                                                        102
 R 00072 00002                                                        103
 S 00073 00003 00005                                                  104
 S 00074 00004 00005                                                  105
 E 00071                                                              106
 V 00005 +70000000+02                                                 107
 K 00005                                                              108
*B 00001                   FØURIER SUBTRACTIØN (16)                   109
 R 00075 00293                                                        110
 R 00067 00002                                                        111
 S 00068 00003 00005                                                  112
 S 00069 00004 00005                                                  113
 F 00066                                                              114
 V 00005 +65000000+02                                                 115
 K 00005                                                              116
*B 00001                   FØURIFR MULTIPLICATIØN (21)                117
 R 00070 00111                                                        118
 R 00062 00002                                                        119
 S 00063 00003 00005                                                  120
 S 00064 00004 00005                                                  121
 F 00061                                                              122
 V 00005 +60000000+02                                                 123
 K 00005                                                              124
*B 00001                   FØURIER K-MULTIPLICATIØN (26)              125
 R 00057 000(                                                         126
```

```
                              K=   00125

 G 00414 00001 00003                                                  127
 S 00059 00004 00005                                                  128
 C 00414 00067 00405 00405                                            129
 H 00001 00004 00066                                                  130
 H 00002 00004 00067                                                  131
 H 00003 00004 00069                                                  132
 F 00002                                                              133
 V 00005 +55000000+02                                                 134
 K 00005                                                              135
*B 00001                       FOURIER TERM EXTRACTION (31)           136
 R 00052 00002                                                        137
 G 00419 00001 00003                                                  138
 S 00054 00004 00005                                                  139
 F 00415                                                              140
 V 00005 +50000000+02                                                 141
 K 00005                                                              142
*B 00001                       FOURIER DIFFERENTIATION (36)           143
 R 00055 00305                                                        144
 R 00047 00002                                                        145
 S 00048 00003 00005                                                  146
 S 00049 00004 00005                                                  147
 F 00046                                                              148
 V 00005 +45000000+02                                                 149
 K 00005                                                              150
*B 00001                       FOURIER INTEGRATION (41)               151
 R 00042 00002                                                        152
 R 00050 00321                                                        153
 S 00043 00003 00005                                                  154
 S 00044 00004 00005                                                  155
 F 00041                                                              156
 V 00005 +40000000+02                                                 157
 K 00005                                                              158
*B 00001                       FOURIER ARGUMENT REPLACEMENT (46)      159
 R 00037 00002                                                        160
 S 00038 00003 00005                                                  161
 S 00039 00004 00005                                                  162
 R 00045 00315                                                        163
 F 00036                                                              164
 V 00005 +35000000+02                                                 165
 K 00035                                                              166
*B 00001                       FOURIER PKG ADAPTER                    167
 A 00013 00015 00015                                                  168
```

```
                                                                  PAGE005
                         K=   00180
 G 00005 00001 00003                                              169
 R 00720 00005                  PATS ADAPTER                      170
 A 00005 00005 00005                                              171
 A 00005 00005 00003                                              172
 R 00006 00012                                                    173
*B 00007                                                          174
 A 00006 00006 00011                                              175
 C 00006 00013 00016                                              176
 A 00003 00003 00011                                              177
 G 00008 00001 00003                                              178
 H 00720 00006 00008            PATS ADAPTER                      179
 C 00005 00003 00007                                              180
 A 00006 00013 00013                                              181
 I 00008 -10000000+01                                             182
*B 00018                                                          183
 A 00008 00008 00011                                              184
 H 00920 00008 00012            PATS ADAPTER                      185
 C 00006 00008 00018                                              186
 F 00010                                                          187
*B 00519                                                          188
 A 00005 00004 00920                                              189
 A 00005 00005 00920                                              190
 R 00006 00012                                                    191
 H 00001 00004 00920                                              192
 C 00920 00012 00009 00009      PATS ADAPTER                      193
 H 00001 00004 00011                                              194
 H 00002 00004 00012                                              195
 H 00003 00004 00014                                              196
 E 00002                                                          197
*B 00009                                                          198
 A 00004 00004 00011                                              199
 A 00006 00006 00011                                              200
 C 00006 00013 00017                                              201
 G 00008 00920 00006            PATS ADAPTER                      202
 H 00001 00004 00008                                              203
 C 00005 00004 00009                                              204
 F 00002                                                          205
 V 00011 +10000000+01                                             206
 V 00012 +00000000+00                                             207
 V 00014 +00505050+08                                             208
 V 00015 +98000000+02                                             209
*B 00016                                                          210
```

```
                                   K=   00180

  R 00920 00015                      PATS ADAPTER
  F 00010
 *B 00017
  S 00004 00004 00006
  H 00001 00004 00015
  F 00002
  K 00020
  V 00006 +00000000+00
  V 00007 +10000000+01
  V 00008 -10000000+01
  V 00009 +20000000+01
  V 00010 -20000000+01
  V 00011 +50000000+00
  V 00012 +67108864+08
  V 00013 +10000000+07
  V 00014 +10000000+05
  V 00015 +10000000+03
  V 00016 +10000000-07
  V 00016 +10000000-10                   CRITERION FOR DROPPING TERMS
  V 00016 +10000000-12
  V 00017 +50000000+02
  V 00018 +40000000+01
  V 00019 +19200000+03                    (19.=    192
 *B 00031
  I 00030 -20000000+01
  A 00030 00030 00009
  G 00049 00500 00030                N*1= NO OF TERMS IN A SERIES
  G 00050 00700 00030                N SUB 2
  G 00051 00501 00030                A*1
  G 00052 00701 00030                B SUB 1
  M 00053 00051 00052
  M 00054 00053 00016
  R 00054 00016
  C 00006 00054 00032 00033
  F 00033
 *B 00032
  S 00055 00006 00054
 *B 00033
  R 00055 00054
  I 00056 -20000000+01
 *B 00034
  A 00056 00056 00009
```

```
                              K=    00200

 I 00057 -20000000+01
*B 00035
 A 00057 00057 00009
 G 00058 00501 00056          A*X
 G 00059 00701 00057          B
 M 00060 00058 00059
 C 00006 00060 00036 00037
*B 00036
 S 00061 00006 00060
 F 00038
*B 00037
 R 00061 00060
*B 00038
 C 00055 00061 00029 00039
*B 00029
 C 00057 00006 00040 00198
 F 00198
*B 00040
 M 00047 00049 00009
 S 00047 00047 00009
 C 00047 00056 00034
 F 00198
 V 00140 +00000000+00
 V 00141 -40000000+01
*B 00039
 A 00140 00140 00009
 A 00141 00141 00018
 G 00062 00502 00056          A*
 C 00062 00006 00041 00042
*B 00041
 R 00063 00007
 R 00064 00062
 F 00043
*B 00042
 R 00063 00008
 S 00064 00006 00062
*B 00043
 D 00065 00064 00013
 A 00066 00065 00012
 S 00067 00066 00012
 M 00068 00067 00013
 S 00069 00064 00068
```

```
                                    K=    00200

 D 00070 00069 00014
 A 00071 00070 00012
 S 00072 00071 00012
 M 00073 00072 00014
 S 00074 00069 00073
 D 00075 00074 00015
 A 00076 00075 00012
 S 00077 00076 00012
 M 00078 00077 00015
 S 00079 00074 00078
 A 00080 00079 00012
 S 00081 00080 00012
 G 00082 00702 00057                B*
 C 00082 00006 00044 00045
*B 00044
 R 00083 00007
 R 00084 00082
 F 00046
*B 00045
 R 00083 00008
 S 00084 00006 00082
*B 00046
 D 00085 00084 00013
 A 00086 00085 00012
 S 00087 00086 00012
 M 00088 00087 00013
 S 00089 00084 00088
 D 00090 00089 00014
 A 00091 00090 00012
 S 00092 00091 00012
 M 00093 00092 00014
 S 00094 00089 00093
 D 00095 00094 00015
 A 00096 00095 00012
 S 00097 00096 00012
 M 00098 00097 00015
 S 00099 00094 00098
 A 00100 00099 00012
 S 00101 00100 00012
 S 00102 00072 00017
 S 00103 00077 00017
 S 00104 00061 00017
```

PAGE009

K= 00200

```
 S 00105 00092 00017
 S 00106 00097 00017
 S 00107 00101 00017
 A 00108 00102 00105
 A 00109 00103 00106
 A 00110 00104 00107
 S 00111 00102 00105
 S 00112 00103 00106
 S 00113 00104 00107
 A 00114 00067 00087
 R 00160 00007
 R 00200 00007
 C 00114 00006 00154 00154
 C 00108 00006 00154 00153
 C 00109 00006 00154 00153
 C 00110 00006 00154 00153
 R 00200 00006
 F 00154
*B 00153
 S 00108 00006 00108
 S 00109 00006 00109
 S 00110 00006 00110
 R 00160 00008
*B 00154
 S 00115 00067 00087
 R 00116 00007
 R 00372 00007
 C 00115 00006 00020 00021
 C 00111 00006 00020 00021
 C 00112 00006 00020 00021
 C 00113 00006 00020 00021
 R 00372 00006
 F 00020
*B 00021
 S 00111 00006 00111
 S 00112 00006 00112
 S 00113 00006 00113
 S 00115 00006 00115
 R 00116 00008
*B 00020
 A 00117 00108 00017
 A 00118 00109 00017
```

```
                          K=    00200

A 00119 00110 00017                                           379
A 00120 00111 00017                                           380
A 00121 00112 00017                                           381
A 00122 00113 00017                                           382
V 00495 +99000000+02
C 00114 00495 00198          I+I GREATER THAN 99 END
C 00117 00495 00198          J+J+50 GREATER THAN 99 END
C 00118 00495 00198          K+K+50 GREATER THAN 99 END
C 00119 00495 00198          L+L+50 GREATER THAN 99 END
C 00120 00495 00198          J-J+50 GREATER THAN 99 END
C 00121 00495 00198          K-K+50 GREATER THAN 99 END
C 00122 00495 00198          L-L+50 GREATER THAN 99 END
C 00007 00117 00198          J+J+50 LESS THAN 1 END
C 00007 00118 00198          K+K+50 LESS THAN 1 END
C 00007 00119 00198          L+L+50 LESS THAN 1 END
C 00007 00120 00198          J-J+50 LESS THAN 1 END
C 00007 00121 00198          K-K+50 LESS THAN 1 END
C 00007 00122 00198          L-L+50 LESS THAN 1 END
M 00123 00114 00013                                           383
M 00124 00117 00014                                           384
M 00125 00118 00015                                           385
A 00126 00125 00119                                           386
A 00127 00126 00124                                           387
A 00128 00127 00123                                           388
M 00129 00115 00013                                           389
M 00130 00120 00014                                           390
M 00131 00121 00015                                           391
A 00132 00131 00122                                           392
A 00133 00132 00130                                           393
A 00134 00133 00129                                           394
C 00063 00006 00022 00023                                     395
F 00198                                                       396
*B 00023                                                      397
C 00083 00006 00024 00025                                     398
F 00198                                                       399
*B 00025                                                      400
R 00135 00060                                                 401
M 00136 00135 00011                                           402
R 00137 00128                                                 403
R 00138 00136                                                 404
R 00139 00134                                                 405
S 00136 00006 00136                                           406
```

PAGE011

K=   00200

```
 F 00028
*B 00024
 R 00135 00060
 M 00136 00135 00011
 S 00137 00006 00128
 R 00138 00136
 S 00139 00006 00134
 M 00138 00138 00116
 M 00136 00136 00160
 M 00136 00136 00200
 M 00138 00138 00372
 F 00028
*B 00022
 C 00083 00006 00026 00027
 F 00198
*B 00026
 R 00135 00060
 M 00136 00135 00011
 R 00137 00128
 R 00138 00136
 R 00139 00134
 F 00028
*B 00027
 R 00135 00060
 M 00136 00135 00011
 S 00137 00006 00128
 S 00138 00006 00136
 S 00139 00006 00134
 M 00138 00138 00116
 M 00136 00136 00160
 M 00136 00136 00200
 M 00138 00138 00372
*B 00028
 R 00161 00136
 R 00162 00137
 F 00159
*B 00156
 R 00161 00138
 R 00162 00139
*B 00159
 I 00164 +00000000+00
 I 00165 -10000000+01
```

```
                                K=   00200

 C 00161 00006 00155 00155
 F 00158
*B 00155
 A 00164 00164 00009
 G 00166 00900 00164          STORED A*
 A 00165 00165 00007
 C 00162 00166 00157 00157
 G 00167 00899 00164          STORED A
 A 00168 00167 00161
 C 00168 00006 00201 00201
 H 00899 00164 00006          REPLACE STORED A BY ZERO
 F 00158
*B 00201
 H 00899 00164 00168          STORE (AX+AS)
 E 00158
*B 00157
 C 00900 00165 00155
 A 00900 00900 00007          UPDATE N
 H 00899 00164 00161          STORE AX
 H 00900 00164 00162          STORE A*X
 V 00360 +19600000+03
 C 00900 00360 00198
*B 00158
 R 00177 00156
 R 00156 00178
 R 00178 00177
 F 00177
*B 00178
 M 00151 00050 00009
 S 00151 00151 00009
 C 00151 00057 00035
 M 00152 00049 00009
 S 00152 00152 00009
 C 00152 00056 00034
*B 00198
 M 00179 00900 00009
 I 00197 +00000000+00
 S 00197 00179 00009
 I 00183 +00000000+00
*B 00180
 A 00183 00183 00009
 S 00184 00183 00009
```

```
                                     K=   00200

 G 00185 00899 00183
 G 00186 00900 00183
*B 00181
 A 00184 00184 00009
 G 00187 00901 00184
 G 00188 00902 00184
 C 00006 00185 00193
 R 00191 00185
*B 00194
 C 00006 00187 00195
 R 00192 00187
*B 00196
 C 00191 00192 00182 00190
 E 00182
*B 00190
 H 00901 00184 00185          INTERCHANGE A1+AX
 H 00902 00184 00186          INTERCHANGF A*1+A*X
 R 00185 00187
 R 00186 00188
*B 00182
 C 00197 00184 00181
 H 00899 00183 00185          STØRE LARGEST A
 H 00900 00183 00186          STØRE LARGFST A*
 C 00183 00019 00499
 C 00197 00183 00180 00499    2N-2 GREATER THAN 183 -180
 F 00499
 R 00900 00017
 R 00361 00008                (361)=-1 IND. ØVERFLØW
 F 00499
*B 00193
 S 00191 00006 00185
 F 00194
*B 00195
 S 00192 00006 00187
 E 00196
        FØURIER ADD + SUBTRACT
*B 00209
 I 00207 +10000000+01
 F 00210
*B 00208
 I 00207 -10000000+01
*B 00210
```

```
                        K=   00200

 M 00218 00500 00009           2N*1
 M 00219 00700 00009           2N SUB 2
 I 00900 +00000000+00          N SUB 3 CØUNTER
 I 00223 +00000000+00          BTRNST CØUNTER
 I 00221 +00000000+00          STØRAGE CØUNTER
*B 00211
 A 00223 00223 00009
 G 00224 00699 00223           B
 G 00225 00700 00223           B*
 M 00224 00224 00207           +B FØR ADD
 H 00699 00223 00224           STØRE B
 H 00700 00223 00225           STØRE B*
 C 00219 00223 00211
 I 00220 +00000000+00          (220)= A CØUNTER
*B 00212
 A 00220 00220 00009
 G 00226 00499 00220           A
 G 00227 00500 00220           A**
 I 00222 +00000000+00          B CØUNTER = 0 ØRIGINALY
*B 00213
 A 00222 00222 00009           UPDATE B CØUNTER BY 2
 G 00228 00699 00222                  BX
 G 00229 00700 00222           B*
 C 00227 00229 00214 00214     A*=B* GØ TØ 214
 A 00226 00226 00228
 H 00699 00222 00006
 H 00700 00222 00006
 C 00226 00006 00214 00214
 F 00215
*B 00214
 C 00219 00222 00213           2N
 C 00226 00006 00483 00483
 F 00215
*B 00483
 A 00221 00221 00009           UPDATE STØR. CTR. BY 2
 A 00900 00900 00007
 H 00899 00221 00226           STØRE A
 H 00900 00221 00227           STØRF A*
*B 00215
 C 00218 00220 00212           2N*1 GET A*X+1
*B 00216
 I 00230 +00000000+00          (230)= CØUNTER FØR B SERIES STILL STØ2
```

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```
                         K=   00200

*B 00217
 A 00230 00230 00009          (230)+2
 G 00231 00699 00230          B
 G 00232 00700 00230          B*
 C 00231 00006 00235 00235    B=0 GO TO 35
 F 00236
*B 00235
 A 00221 00221 00009          UPDATE TER + ST. CTN.
 A 00900 00900 00007          UPDATE N SUB 3 COUNTER
 H 00899 00221 00231          STORF B
 H 00900 00221 00232          STORE B*
*B 00236
 C 00219 00230 00217          2N*2) 230 GO TO 217
 E 00198
        FOURIER  DIFFERENTIAL
*B 00240
 I 00247 +00000000+00
 I 00248 +00000000+00
 I 00900 +00000000+00         N
 M 00255 00700 00009          2N
*B 00241
 A 00247 00247 00009
 R 00256 00007
 G 00249 00699 00247          A
 G 00250 00700 00247          A*
 C 00250 00006 00246
 S 00250 0000( 00250
 R 00256 00008
*B 00246
 D 00251 00250 00013
 A 00251 00251 00012
 S 00251 00251 00012
 C 00251 00006 00257
 F 00244
*B 00257
 C 00256 00006 00242 00243
*B 00243
 M 00252 00249 00251
 R 00253 00250
 F 00245
*B 00242
 M 00252 00249 00251          -IA
```

K= 00200

```
 M 00252 00252 00008
 M 00253 00250 00008
*B 00245
 A 00248 00248 00009          B+2
 A 00900 00900 00007          UPDATE N COUNTER
 H 00899 00248 00252          STØRE A
 H 00900 00248 00253          STØRE A*
*B 00244
 C 00255 00247 00241
 E 00198
        FØURIER BAR, INT. SCALAR MULTIPLY BY A CØNSTANT AND EXTRACT
*B 00260
 R 00276 00007                (276)=1 BAR ØPERATIØN
 E 00262
*B 00261
 R 00276 00006                (276)=0 INTEGRATE
*B 00262
 M 00277 00700 00009             2N
 I 00900 +00000000+00
 I 00278 +00000000+00         278 = SERIES CØUNTER
 I 00279 +00000000+00         279 = STØRAGE CØUNTER
*B 00263
 R 00258 00007                (258) = SIN CØS IND
 A 00278 00278 00009          UPDATE (278)
 G 00280 00699 00278          A
 G 00281 00700 00278          A*
 C 00281 00006 00264          A* + GØ TØ 264
 S 00281 00006 00281          -A* TØ +A*
 R 00258 00008                (258)=-1 SIN INDICATØR
*B 00264
 D 00282 00281 00013          A*/10 6
 A 00283 00282 00012
 S 00283 00283 00012          +1I1
 M 00284 00283 00013          1XX1001001001
 S 00285 00281 00284          XX1XX1XX1
 D 00286 00285 00014          XX.XXXX
 A 00287 00286 00012
 S 00287 00287 00012           J + ØR - 50
 M 00288 00287 00014            XX0000
 S 00289 00285 00288            XXXX
 D 00290 00289 00015               XX.XX
 A 00291 00290 00012
```

```
                               K=   00200

 S 00291 00291 00012              + ØR -K +50                          659
 M 00292 00291 00015                                                   660
 S 00293 00289 00292                                                   661
 A 00294 00293 00012                                                   662
 S 00294 00294 00012              + ØR -L +50                          663
 S 00296 00287 00017             J                                     664
 S 00297 00291 00017             K                                     665
 S 00298 00294 00017             L                                     666
 C 00276 00006 00265             BAR 265, INT 266                      667
 F 00266                                                               668
*B 00265                                                               669
 A 00300 00283 00296             I+J=J*                                670
 C 00300 00006 00267 00268       J=0                                   671
 C 00297 00006 00267 00268       K=0                                   672
 C 00298 00006 00267 00268       L=0                                   673
 M 00282 00282 00258             J=0,K=0,L=0, SIN ØR CØS               674
 R 00312 00007                   (312)=+1 SIGN CHANGE INDICATØR        675
 C 00282 00006 00267 00270       CØS(0,0,0,0) 267, SIN(0,0,0,0) ØMIT TERM  676
*B 00268                                                               677
 S 00300 00006 00300             J 1 (-1)                              678
 S 00297 00006 00297             K (-1)                                679
 S 00298 00006 00298             L (-1)                                680
 R 00312 00008                   (312)=-1 SIGN CHANGE INDICATØR        681
 C 00258 00006 00267                                                   682
 M 00280 00280 00312             SIN TERM CHANGE TØ -A                 683
*B 00267                                                               684
 A 00301 00300 00017             J 1+50                                685
 C 00301 00495 00198             1+J+50 GREATER THAN 99 END
 A 00302 00297 00017             K+50                                  686
 A 00303 00298 00017             L+50                                  687
 M 00304 00302 00015             10 2K                                 688
 M 00305 00301 00014             10 4 J 1                              689
 A 00306 00304 00303                                                   690
 A 00307 00306 00305             A* ARS. VALUF                         691
 M 00307 00307 00258             (A* ØR -A* IF SIN ØR CØS              692
*B 00269                                                               693
 A 00900 00900 00007             UPDATE N CØUNTER                      694
 A 00279 00279 00009             UPDATE STØRAGE CØUNTER                695
 H 00899 00279 00280             A                                     696
 H 00900 00279 00307             A*                                    697
*B 00270                                                               698
 C 00277 00278 00263             2N GREATER 278 LØØP GET N+1 (A)(A*)   699
```

K=   00200

```
 F 00198                                                  700
 V 00321 +20000000+00                                     701
 V 00322 +20000000+00                                     702
*B 00266                          INTEGRATE               703
 C 00296 00006 00313 00313        J=0 T⊙ 313              704
 C 00297 00006 00314 00314        J=0 IS K=0              705
 R 00368 00008                    368=-1                  706
 R 00369 00280                    A                       707
 M 00370 00281 00258              I A*                    708
 E 00270                                                  709
*B 00314                          J=0,K=0                 710
 M 00308 00297 00322              KG                      711
 D 00309 00280 00308              A/KG                    712
 C 00258 00006 00271 00272        SIN ⊙R C⊙S              713
*B 00313                                                  714
 M 00305 00296 00321              JC                      715
 M 00306 00297 00322              KG                      716
 A 00308 00306 00305              JC+KG                   717
 C 00308 00006 00315 00315        JC+KG=0                 718
 E 00270                          IND. ⊙N PRINT ⊙UT BUT C⊙NTINUE 719
*B 00315                                                  720
 D 00309 00280 00308              A/JC+KG                 721
 C 00258 00006 00271 00272        SIN OR C⊙S              722
*B 00271                                                  723
 R 00280 00309                    A/JC+KG OR A/KG         724
 M 00307 00281 00008              -A*                     725
 F 00269                                                  726
*B 00272                                                  727
 M 00280 00309 00008              -A/JC+KG ⊙R -A/KG       728
 R 00307 00281                    A*                      729
 E 00269                                                  730
 V 00339 +20000000+00             (339)= C⊙NSTANT         731
*B 00330                          C⊙NSTANT MULTIPLY       732
 I 00331 +00000000+00             (331)= ST⊙RAGE C⊙UNTER  733
 R 00900 00500                                            734
 A 00333 00500 00500                                      735
*B 00334                                                  736
 A 00331 00331 00009              (331)+2                 737
 G 00335 00499 00331                                      738
 G 00336 00500 00331                                      739
 M 00337 00335 00339              CA                      740
 H 00899 00331 00337              ST⊙RE A                 741
```

```
                                                                              PAGE019
                                      K=   00200

  H 00900 00331 00336                    STORE A*                             742
  C 00333 00331 00334                    2N GREATER THAN (631)                743
  F 00499                                                                     744
 *B 00345                                                                     745
  V 00349 +00505050+08                   ARGUMENT OF TERM FOR EXTRACTION      746
  M 00350 00500 00009                    2N                                   747
  I 00351 +00000000+00                   (351)=0 INITIAL                      748
 *B 00346                                                                     749
  A 00351 00351 00009                    (351)+2                              750
  G 00352 00500 00351                                                         751
  C 00352 00349 00347 00347              2N=(351)                             752
  G 00353 00499 00351                                                         753
  R 00901 00353                                                               754
  R 00902 00352                                                               755
  R 00900 00007                                                               756
  F 00499                                                                     757
 *B 00347                                                                     758
  C 00350 00351 00346                      2N=(351)                           759
  R 00900 00006                                                               760
  F 00499                                                                     761

                                                                          0777 CARDS
```

Appendix D

Mystic Code

The Fourier Operating Package described in this report was written for the IBM 709 in Mystic Code. The following is an explanation of the Mystic Code.

```
                                                  MYSTIC CODE           PAGE001
                         K=   00000

C ODE               REMARKS                                            06/20/61

A  X Y  Z           ADD COMMAND (Y) + (Z) GO INTO LOCATION X. THAT IS,
                    ADD (Y) TO (Z) AND STORE THE RESULT IN X.

*B  X               BEGIN POINT PSEUDO COMMAND. THE SEQUENCE OF INSTRUCTIONS
                     WHICH FOLLOW A BEGIN X IS ENTERED
                     FROM AT LEAST TWO PARTS OF THE CODING. ENTRANCE TO
                     A BEGIN INSTRUCTION CAN  BE FROM THE INSTRUCTION
                     PRECEDING IT (I.E.,A SEQUENTIAL ENTRANCE), FROM
                     A COMPARE COMMAND OR FROM AN END COMMAND. THIS
                     INSTRUCTION MAY BE THOUGHT OF AS STATING THAT THE
                     FOLLOWING INSTRUCTIONS MAY BE ENTERED BY A TRANSFER
                     TO LOCATION X.

C  X Y  Z           COMPARE (X) WITH (Y). IF (X) IS GREATER THAN (Y),GO TO
                     LOCATION Z AND EXECUTE THE INSTRUCTIONS WHICH FOLLOW
                     THE B Z OR N Z. IF (X) EQUALS OR IS LESS THAN (Y),
                     EXECUTE THE NEXT INSTRUCTION IN SEQUENCE.

C  X Y  Z Z*        COMPARE (X) WITH (Y). IF (X) IS GREATER THAN (Y), GO TO
                     LOCATION Z AND EXECUTE THE INSTRUCTIONS WHICH FOLLOW
                     THE B Z OR N Z. IF (X) IS LESS THAN (Y),GO TO LOCATION
                     Z* AND EXECUTE THE INSTRUCTIONS WHICH FOLLOW THE B Z*
                     OR N Z*. IF (X) EQUALS (Y), EXECUTE THE NEXT INSTRUC-
                     TION IN SEQUENCE.

D  X Y  Z           DIVIDE COMMAND. (Y)/(Z) GO INTO LOCATION X.
                     THAT IS, DIVIDE (Y) BY (Z) AND STORE THE RESULT IN X.

E  X                EXIT COMMAND. UNCONDITIONAL TRANSFER TO
                     THE SET OF INSTRUCTIONS WHICH FOLLOW B X OR
                     N X.

F  X Y  Z           FUNCTION COMMAND.
                     THIS INSTRUCTION ENABLES ONE TO TRANSFER TO A
                     FUNCTION AND AFTER ITS EXECUTION, CONTINUE TO
                     THE NEXT INSTRUCTION IN SEQUENCE FOLLOWING THE
                     FUNCTION STATEMENT. Y IS THE LOCATION OF THE
                     FIRST BEGIN INSTRUCTION (B 1) IN A FUNCTION.
                     NORMALLY Z IS THE LOCATION OF THE INPUT TO THE
```

```
                 K=   00000

                        FUNCTION,WHILE X IS THE LOCATION OF THE
                        OUTPUT. HOWEVER, THE REQUIREMENTS OF
                        FUNCTIONS DIFFER FROM FUNCTION TO FUNCTION AND
                        EACH ONE USED MUST HAVE ITS REQUIREMENTS MET
                        BEFORE TRANSFERRING TO IT. (NOTE. FUNCTIONS ARE
                        SOMETIMES REFERRED TO AS SUB-ROUTINES.)

G  X Y  Z              GET COMMAND. THE CONTENTS OF A LOCATION EQUAL
                        TO (Z) + Y GO IN TO LOCATION X. THAT IS, REPLACE
                        THE CONTENTS OF LOCATION X BY THE CONTENTS OF
                        Y + (Z).

H  X Y  Z              HOLD COMMAND. THE CONTENTS OF LOCATION Z GO INTO
                        A LOCATION = X + (Y) . THAT IS, REPLACE THE CONTENTS
                        OF X + (Y) BY THE CONTENTS OF Z.

I  X SY YYYYYYYSEE     INITIALIZE COMMAND. SET THE CONTENTS OF
                        X = S.YYYYYYYY TIMES 10 EXP SEE. S= BLANK OR +
                        MEANS POSITIVE VALUE, S= - MEANS NEGATIVE VALUE IN
                        THE MANTISSA. S IN THE EXPONENT USES THE SAME
                        SIGN NOTATION.

K  X                   KEY ADDRESS PSEUDO COMMAND.
                        X=0 , SET THE K COUNTER TO ZERO. WHEN X DOES NOT
                        EQUAL ZERO, THE KEY ADDRESS IS INCREASED BY X.
                        THAT IS, THE K COUNTER + X GOES INTO THE K
                        COUNTER. THE K COUNTER IS USED TO RELOCATE
                        FUNCTIONS. THE K COUNTER IS ADDED TO THE ADDRESS
                        OF EACH INSTRUCTION DURING COMPILATION, UNLESS
                        THE ADDRESS IS IN THE LEFT HAND ADDRESS OF A
                        Q COMMAND. THE K COMMAND CLEARS THE Q TABLE
                        DURING COMPILATION.

L  X Y  Z  F1 ,F2,. ...F18,T1,T2,...T18.
                       Z= CA ,LOAD  F1 COLUMNS OF DATA INTO LOCATION X,THEN
                              LOAD  F2 COLUMNS OF DATA INTO LOCATION X+1,
                              ETC. TO MAXIMUM OF 18 FIELDS OR 72 COLUMNS.
                              T1,T2,..T18 DESCRIBE THE WAY THE DATA IS
                              STORED IN X,X+1,ETC. THE TYPE OF DATA STORED
                              IS NUMERIC OR ALPHABETIC. IF IT IS NUMERIC,
                              THE CORRESPONDING TYPE CODE IS N. IF IT IS
                              ALPHABETIC, THE CORRESPONDING TYPE CODE IS
```

```
K=   07000

          A. TYPE OF DATA CODE ALSO INCLUDES S FOR
          SKIP. WHEN THE SKIP TYPE IS USED, THIS
          DOES NOT INVOLVE DATA STORED IN A WORD.
          THUS, IF T1 IS TYPE S, T2 IS TYPE N, THE
          L COMMAND READS THE DATA FROM THE F2 COLUMN
          INTO WORD X. THE COLUMNS ARE READ STARTING
          FROM THE LEFT ALWAYS. SKIP FIELDS TO THE RIGHT
          OF ALL OTHER DATA NEED NOT BE DEFINED IN THE
          L COMMAND. THE FIRST BLANK FIELD MEANS THE REST
          OF THE CARD IS SKIPPED. THE NUMBER OF COLUMNS
          OF TYPE A MUST NOT EXCEED 4 PER FIELD. THE
          NUMBER OF COLUMNS OF TYPE N MUST NOT EXCEED 9
          PER FIELD INCLUDING THE SIGN. THE NUMBER OF
          COLUMNS OF TYPE S MUST NOT EXCEED 15 PER FIELD.
          IF 1 CARD IS TO BE LOADED ACCORDING TO THE
          GIVEN FORMAT, (Y) = 1. IF I CARDS ARE TO BE
          LOADED ACCORDING TO THE GIVEN FORMAT,
          (Y)= I,THE NUMBER OF CARDS. THE DATA FOR
          EACH CARD IS CONSECUTIVELY STORED IN THE SAME
          WAY AS THAT FOR THE FIRST. THE DATA FOR THE
          FIRST WORD OF INPUT FROM CARD J+1 FOLLOWS
          CONSECUTIVELY THE LAST WORD OF INPUT FROM
          CARD J. (Y) MUST BE AN INTEGER THAT IS AT LEAST 1.
       Z= TA ,LOAD  TAPE B1 BCD.     SAME FORMAT DESCRIPTION
       Z= TB ,LOAD  TAPE B2 BCD.      AS ABOVE IN Z= CA,
       Z= TC ,LOAD  TAPE B3 BCD.      REPLACING CARD BY BCD
       Z= TD ,LOAD  TAPE B4 BCD.      RECORD AND LOAD BY
       Z= TE ,LOAD  TAPE B5 BCD.      THE APPROPIATE READ
       Z= TF ,LOAD  TAPE B6 BCD.      TAPE DESIGNATION.
       Z= TG ,LOAD  TAPE B7 BCD.
       Z= TA ,(Y) =I , BLANK F1 THROUGH F18 AND T1 TO T18.
       Z= TB ,(Y) =I , BLANK F1 THROUGH F18 AND T1 TO T18.
       Z= TC ,(Y) =I , BLANK F1 THROUGH F18 AND T1 TO T18.
       Z= TD ,(Y) =I , BLANK F1 THROUGH F18 AND T1 TO T18.
       Z= TE ,(Y) =I , BLANK F1 THROUGH F18 AND T1 TO T18.
       Z= TF ,(Y) =I , BLANK F1 THROUGH F18 AND T1 TO T18.
       Z= TG ,(Y) =I , BLANK F1 THROUGH F18 AND T1 TO T18.
    IF I = 0,   THE COMMAND IS TO BACKSPACE A FILE.
    IF I = -N   THE COMMAND IS TO BACKSPACE N RECORDS.

L  X Y  Z      Z= CAB, LOAD INTO X,X+1,ETC. BINARY CARDS
                   HAVING THE NUMBER OF WORDS SPECIFIED
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                         K=   07000

                              IN THE CONTENTS OF Y.
                         Z= TAB, LOAD TAPE B1 BINARY.       LOAD INTO X,X+1,ETC.
                         Z= TBB, LOAD TAPE B2 BINARY.        A BINARY RECORD
                         Z= TCB, LOAD TAPE B3 BINARY.        HAVING THE NUMBER
                         Z= TDB, LOAD TAPE B4 BINARY.        OF WORDS SPECIFIED
                         Z= TEB, LOAD TAPE B5 BINARY.        IN THE CONTENTS OF
                         Z= TFB, LOAD TAPE B6 BINARY.        Y.
                         Z= TGB, LOAD TAPE B7 BINARY.

M  X Y  Z                MULTIPLY COMMAND. (Y) TIMES (Z) GOES INTO X.
                          THAT IS, THE PRODUCT (Y)(Z) REPLACES THE
                          CONTENTS OF X.

N  X                     NOTE COMMAND. NOTE THAT THE FOLLOWING COMMAND
                          SEQUENCE BEGINS WITH X. N X IS NORMALLY USED
                          TO BEGIN SEVERAL COMMAND SEQUENCES. THE N X
                          EXECUTED LATEST IS THE COMMAND SEQUENCE TO
                          BE FOLLOWED WHEN AN UNCONDITIONAL TRANSFER
                          OR A CONDITIONAL TRANSFER TO X IS MADE. THE N X
                          IS A VARIABLE CONNECTOR.

O  X                     ORIGIN COMMAND. THE COMPILATION WILL GENERATE
                          MACHINE LANGUAGE CODE FROM LOCATION X TO A
                          MAXIMUM OF 30,000.

P  X Y  Z  F1 ,F2,. .. F18,T1,T2,...T18.
                         Z= CA ,PUNCH F1 COLUMNS OF DATA FROM LOCATION X,THEN
                                PUNCH F2 COLUMNS OF DATA FROM LOCATION X+1,
                                ETC. TO MAXIMUM OF 18 FIELDS OR 72 COLUMNS.
                                T1, T2,..T18 DESCRIBE THE WAY THE DATA IS
                                STORED IN X,X+1,ETC. THE TYPE OF DATA STORED
                                IS NUMERIC OR ALPHABETIC. IF IT IS NUMERIC,
                                THE CORRESPONDING TYPE CODE IS N. IF IT IS
                                ALPHABETIC, THE CORRESPONDIND TYPE CODE IS
                                A. TYPE OF DATA CODE ALSO INCLUDES S FOR
                                SKIP. WHEN THE SKIP TYPE IS USED, THIS
                                DOES NOT INVOLVE DATA STORED IN A WORD.
                                THUS, IF T1 IS TYPE S, T2 IS TYPE N, THE
                                P COMMAND GETS THE DATA FOR THE F2 COLUMNS
                                FROM WORD X. THE COLUMNS ARE BUILT UP STARTING
                                FROM THE LEFT ALWAYS. SKIP FILEDS TO THE RIGHT
                                OF ALL OTHER DATA NEED NOT BE DEFINED IN THE
```

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K=   07000

            P COMMAND. THE FIRST BLANK FIELD MEANS THE REST
            OF THE CARD IS SKIPPED. THE NUMBER OF COLUMNS
            OF TYPE A MUST NOT EXCEED 4 PER FIELD. THE
            NUMBER OF COLUMNS OF TYPE N MUST EXCEED 9
            PER FIELD INCLUDING THE SIGN. THE NUMBER OF
            COLUMNS OF TYPE S  MUST NOT EXCEED 15 PER FIELD.
            IF 1 CARD IS TO BE PUNCHED ACCORDING TO THE
            GIVEN FORMAT, (Y) = 1. IF I CARDS ARE TO BE
            PUNCHED ACCORDING TO THE GIVEN FORMAT,
            (Y)= I,THE NUMBER OF CARDS. THE DATA FOR EACH
            CARD MUST BE CONSECUTIVELY STORED IN THE SAME
            WAY AS THAT FOR THE FIRST. THE DATA FOR THE
            FIRST WORD OF OUTPUT FOR CARD J+1 MUST FOLLOW
            CONSECUTIVELY THE LAST WORD OF OUTPUT FOR
            CARD J. (Y) MUST BE AN INTEGER THAT IS AT LEAST
        Z= PA ,PRINT.  INSERT PRINT FOR PUNCH IN THE DESCRIP-
            TION ABOVE TO INTERPRET AN INSTRUCTION TO PRINT
            ON THE ON-LINE PRINTER. LINE REPLACES CARD IN
            THE EXPLANATION ABOVE.
        Z= TA ,WRITE TAPE B1 BCD.     SAME FORMAT DESCRIPTION
        Z= TB ,WRITE TAPE B2 BCD.      AS ABOVE IN Z= CA,
        Z= TC ,WRITE TAPE B3 BCD.      REPLACING CARD BY BCD
        Z= TD ,WRITE TAPE B4 BCD.      RECORD AND PUNCH BY
        Z= TE ,WRITE TAPE B5 BCD.      THE APPROPIATE WRITE
        Z= TF ,WRITE TAPE B6 BCD.      TAPE DESIGNATION.
        Z= TG ,WRITE TAPE B7 BCD.
        Z= TA ,(Y) =I , BLANK F1 THROUGH F18 AND T1 TO T18.
        Z= TB ,(Y) =I , BLANK F1 THROUGH F18 AND T1 TO T18.
        Z= TC ,(Y) =I , BLANK F1 THROUGH F18 AND T1 TO T18.
        Z= TD ,(Y) =I , BLANK F1 THROUGH F18 AND T1 TO T18.
        Z= TE ,(Y) =I , BLANK F1 THROUGH F18 AND T1 TO T18.
        Z= TF ,(Y) =I , BLANK F1 THROUGH F18 AND T1 TO T18.
        Z= TG ,(Y) =I , BLANK F1 THROUGH F18 AND T1 TO T18.
    IF I = 0,   THE COMMAND IS TO WRITE AN END OF FILE MARK
            ON THE APPROPIATE TAPE.
    IF I = -1,  THE COMMAND IS TO REWIND THE APPROPIATE TAPE.

P  X Y  Z       Z= CAB, PUNCH FROM X,X+1,ETC. UP TO THE NUMBER OF
                     WORDS SPECIFIED IN (Y) TO CARDS IN BINARY
                     FORM.
                Z= TAB, WRITE TAPE B1 BINARY.      WRITE FROM X,X+1,
                Z= TBB, WRITE TAPE B2 BINARY.       ETC. UP TO THE
```

```
                 K=   07000

              Z= TCB, WRITE TAPE B3 BINARY.      NUMBER OF WORDS
              Z= TDB, WRITE TAPE B4 BINARY.      SPECIFIED IN THE
              Z= TEB, WRITE TAPE B5 BINARY.      CONTENTS OF Y.
              Z= TFB, WRITE TAPE B6 BINARY.
              Z= TGB, WRITE TAPE B7 BINARY.

Q  X Y        THE Q COMMAND ENABLES ONE TO FIX ADDRESSES. THE
              K COUNTER MODIFIES EVERY ADDRESS EXCEPT EACH ONE
              EQUAL TO AN X IN A Q COMMAND. EACH SUCH ADDRESS
              IS THEN CHANGED DURING COMPILATION TO THE ADDRESS
              GIVEN IN THE Y ADDRESS OF THE Q COMMAND. THE
              Q TABLE IS CLEARED BY A K COMMAND DURING COMPILATION.
              THIS ENABLES ONE TO HAVE A SEPARATE Q TABLE FOR EACH
              FUNCTION. THE Q COMMAND MUST PRECEDE THE INSTRUCTIONS
              IT IS TO CONTROL. IT IS GOOD PRACTICE TO HAVE THE
              Q COMMANDS PRECEDE ANY OTHER INSTRUCTIONS WHICH
              FOLLOW A K COMMAND.

R  X Y        REPLACE COMMAND.
               REPLACE THE CONTENTS OF X BY THE CONTENTS OF Y.

S  X Y  Z     SUBTRACT COMMAND. REPLACE THE CONTENTS OF X BY
               (Y) - (Z).

   T XX...X   TITLE COMMAND. THE CHARACTERS IN A TITLE COMMAND MAY
               GO FROM COL.2 TO COL.71. THEY CONTROL COL.
               1 TO COL.70 OF AN OUTPUT CARD, PRINTER LINE, OR
               BCD LISTED LINE. A TITLE COMMAND NEED NOT PRECEDE
               EACH P COMMAND. THE LATEST T EXECUTED IS THE
               ONE IN POWER. IF NO TITLE INFORMATION IS DESIRED
               FOR THE OUTPUT, THE T COMMAND SHOULD HAVE COL.
               2 TO 71 BLANK. IF TITLE INFORMATION IS DESIRED,
               IF SHOULD BE ARRANGED SO THAT IT WILL CONTROL
               ONLY BLANK COLUMNS OF THE P COMMAND.

V  X SY YYYYYYYSEE   VALUE PSEUDO COMMAND. THE VALUE REPRESENTED AS A NORMALIZED
               FLOATING POINT NUMBER IS STORED IN X DURING
               COMPILATION. THE VALUE COMMAND IS EXECUTED ONLY
               DURING COMPILING. SEE INITIALIZE COMMAND FOR FORMAT
               OF VALUF.

W  X YY YY    WORD PSEUDO COMMAND.  THE WORD YYYY IS STORED IN ALPHABETIC
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                                        MYSTIC CODE            PAGE007
         K=    07000

CODE IN LOCATION X. YYYY MAY BE NUMERIC OR ALPHABETIC
CHARACTERS.

                                                               0255 CARDS
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